

## Combining uncertainties in several quantities: adding or subtracting

When one *adds or subtracts* several measurements together, one simply adds together the uncertainties to find the uncertainty in the sum.

Rob is 186 +/- 2 cm tall, and Jane is 147 +/- 3 cm tall. What is their combined height?

$$\begin{aligned}\text{combined height} &= 186 \text{ cm} + 147 \text{ cm} \\ &= 333 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{uncertainty in combined height} &= 2 \text{ cm} + 3 \text{ cm} \\ &= 5 \text{ cm}\end{aligned}$$

$$\text{combined height} = 333 \text{ cm} \pm 5 \text{ cm}$$

**Orally:** Now, if all the quantities have roughly the same magnitude and uncertainty -- as in the example above -- the result makes perfect sense. But if one tries to add together very different quantities, one ends up with a funny-looking uncertainty.

e.g., Using a pair of calipers, Rob measures a flea to have a height of 0.020 cm +/- 0.003 cm. What is their combined height? If we follow the rules, we find

$$\begin{aligned}\text{combined height} &= 186 \text{ cm} + 0.020 \text{ cm} \\ &= 186.020 \text{ cm} \sim 186 \text{ cm}\end{aligned}$$

(because the rule for addition is rounding to the least degree of precision in the estimated digit.)

$$\begin{aligned}\text{uncertainty in combined height} &= 2 \text{ cm} + 0.003 \text{ cm} \\ &= 2.003 \text{ cm} \sim 2 \text{ cm}\end{aligned}$$

(because the uncertainty must match the place value of the estimated digit.)

Therefore the combined height = 186 cm +/- 2 cm



For subtraction, we still add the uncertainties because when we combine two or more measures we do not become more certain in the final answer.

## Combining uncertainties in several quantities: multiplying and dividing

When one *multiplies or divides* several measurements together, one can often determine the percentage uncertainty in the final result simply by adding the uncertainties in the several quantities.

Q: Why convert the reading errors to a percent?

A: Often, multiply and divide yield derived units (i.e., two or more units combined) which make it impossible to add the reading errors. Since percent is unitless, we use this as a means to express the magnitude of the reading error without the units.

1. Jane needs to calculate the volume of her pool, so that she knows how much water she'll need to fill it. She measures the length, width, and height:

$$\text{Percent reading error} = \frac{\text{reading error}}{\text{measurement}} \times 100$$

(we arbitrarily round to the tenths place for the final answer)

$$\text{width } W = 3.12 \pm 0.08 \text{ m}$$

$$\text{e.g., Percent reading error for width} = \frac{0.08}{3.12} \times 100 = 2.5641\%$$

$$W = 3.12 \text{ m} \pm 2.5641\%$$

$$\begin{aligned} \text{length } L &= 5.56 \pm 0.14 \text{ m} \\ &= 5.56 \text{ m} \pm 2.51798\% \end{aligned}$$

(units do not follow the percentage because % is unitless)

$$\begin{aligned} \text{depth } D &= 2.94 \pm 0.11 \text{ m} \\ &= 2.94 \text{ m} \pm 3.74149\% \end{aligned}$$

To calculate the volume, she multiplies together the length, width and depth:

$$\text{volume} = L * W * D = (5.56 \text{ m}) * (3.12 \text{ m}) * (2.94 \text{ m})$$

$$= 51.0 \text{ m}^3 \text{ (3 sig dig's)}$$

## Lesson 7

$$\text{percentage uncertainty in volume} = \begin{aligned} &(\text{percentage uncertainty in L}) + \\ &(\text{percentage uncertainty in W}) + \\ &(\text{percentage uncertainty in D}) \end{aligned}$$

$$= 2.5641\% + 2.51798\% + 3.74149\%$$

$$\sim 8.8\%$$

$$\text{Therefore the volume of the pool} = 51.0 \text{ m}^3 \pm 8.8\%$$

2. Bob wants to calculate the rate of a reaction in g/s of zinc. The following data was collected:

$$\mathbf{15.0 \pm 0.1 \text{ g of Zinc was used in } 135.67 \pm 0.01 \text{ s}}$$

*Orally: The unit g/s tells us that we need to take mass (g) and divide it by time (s).*

$$\text{Therefore: } \frac{15.0 \pm 0.1 \text{ g}}{135.67 \pm 0.01 \text{ s}}$$

Since I can't add the reading errors here, I'm forced to convert them to a percentage.

$$= \frac{15.0 \text{ g} \pm 0.666\%}{135.67 \pm 0.0073\%} = 0.11056 \text{ g/s} \pm 0.674\%$$

$$\sim 0.111 \text{ g/s} \pm 0.7\%$$

(3 sig dig's for the derived value and arbitrary tenths place rounding for the percentage)

## Lesson 7

Try the following excercises:

- 1)  $24.32 \pm 0.01 \text{ g} + 32.46 \pm 0.01 \text{ g}$  (Ans:  $56.78 \pm 0.02 \text{ g}$ )
- 2)  $4.6 \pm 0.2 \text{ }^{\circ}\text{C} - 1.2 \pm 0.1 \text{ }^{\circ}\text{C}$  (Ans:  $3.4 \pm 0.3 \text{ }^{\circ}\text{C}$ )
- 3)  $(20.0 \pm 0.1 \text{ g}) \times (66.4 \pm 0.4 \text{ }^{\circ}\text{C})$  (Ans:  $1330 \pm 1.1\% \text{ g}\cdot^{\circ}\text{C}$ )
- 4)  $13.02 \pm 0.01 \text{ g} + 10.1 \pm 0.5 \text{ g}$  (Ans:  $23.1 \pm 0.5 \text{ g}$ )
- 5)  $7.34 \pm 0.02 \text{ g} \div 3.1 \pm 0.1 \text{ mL}$  (Ans:  $2.4 \pm 3.5\% \text{ g/mL}$ )

Assignment: Pg. 35 Hebden #51, 52 and complete worksheet "Math of Measure"