

Unit 3: Powers & Exponents**3.2B Negative Exponents**

Continue each of the following patterns:

$$\begin{aligned}
 2^4 &= 16 \div 2 \\
 2^3 &= 8 \div 2 \\
 2^2 &= 4 \div 2 \\
 2^1 &= 2 \div 2 \\
 2^0 &= 1 \div 2 \\
 2^{-1} &= \frac{1}{2} \\
 2^{-2} &= \frac{1}{4} \\
 2^{-3} &= \frac{1}{8} \\
 2^{-4} &= \frac{1}{16}
 \end{aligned}$$

$$\begin{aligned}
 3^4 &= 81 \div 3 \\
 3^3 &= 27 \div 3 \\
 3^2 &= 9 \div 3 \\
 3^1 &= 3 \div 3 \\
 3^0 &= 1 \\
 3^{-1} &= \frac{1}{3} = \frac{1}{3^1} \\
 3^{-2} &= \frac{1}{9} = \frac{1}{3^2} \\
 3^{-3} &= \frac{1}{27} = \frac{1}{3^3}
 \end{aligned}$$

$$7^{-6} = \frac{1}{7^6}$$

How does the result of a negative exponent connect to the result of a positive exponent?

★ negative exponents produce reciprocal results

Another approach.....

What exponent law is likely to lead to a negative exponent?

$$\begin{aligned}
 \frac{3^2}{3^5} &= \frac{\cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot 3 \cdot 3 \cdot 3} = \frac{1}{3 \cdot 3 \cdot 3} \\
 &= 3^{2-5} \\
 &= 3^{-3} = \frac{1}{3^3}
 \end{aligned}$$

Rule: $b^{-m} = \frac{1}{b^m}$

Ex. Simplify (write with a single positive exponent) & then evaluate:

a) 5^{-3}

$$\begin{aligned}
 &= \frac{1}{5^3} \\
 &= \frac{1}{125}
 \end{aligned}$$

b) 7^{-1}

$$\begin{aligned}
 &= \frac{1}{7^1} \\
 &= \frac{1}{7}
 \end{aligned}$$

c) 10^{-5}

$$\begin{aligned}
 &= \frac{1}{10^5} \\
 &= \frac{1}{100,000}
 \end{aligned}$$

d) -2^{-3}

$$\begin{aligned}
 &= -\frac{1}{2^3} \\
 &= -\frac{1}{8}
 \end{aligned}$$

e) $(-2)^{-4}$

$$\begin{aligned}
 &= \frac{1}{(-2)^4} \\
 &= \frac{1}{16}
 \end{aligned}$$

f) $-(-2)^{-6}$

$$\begin{aligned}
 &= -\frac{1}{(-2)^6} \\
 &= -\frac{1}{64}
 \end{aligned}$$

g) $\frac{1}{3^{-3}}$

$$\begin{aligned}
 &= \frac{1}{\frac{1}{3^3}} \\
 &= 27
 \end{aligned}$$

h) $\frac{1}{5^{-2}}$

$$\begin{aligned}
 &= 5^2 \\
 &= 25
 \end{aligned}$$

Ex. Simplify using exponent laws:

a) $4^3 \times 4^{-1}$

$$= 4^{3+(-1)}$$

$$= \boxed{4^2}$$

b) 2×2^{-4}

$$= 2^{1+(-4)}$$

$$= 2^{-3}$$

$$= \boxed{\frac{1}{2^3}}$$

c) $3^5 \div 3^8$

$$= 3^{5-8}$$

$$= 3^{-3}$$

$$= \boxed{\frac{1}{3^3}}$$

d) $\frac{9}{9^{-4}}$

$$= 9^{1-(-4)}$$

$$= \boxed{9^5}$$

e) $\frac{6^{-7}}{6^{-2}}$

$$= 6^{-7-(-2)}$$

$$= 6^{-5}$$

$$= \boxed{\frac{1}{6^5}}$$

f) $(5^{-2})^3$

$$= 5^{-6}$$

$$= \frac{1}{5^6}$$

g) $(8^3)^{-1}$

$$= 8^{-3}$$

$$= \frac{1}{8^3}$$

h) $(7^{-3})^{-4}$

$$= \boxed{7^{12}}$$

i) $(5^4 \times 3)^{-2}$

$$= 5^{-2} \times 3^{-2}$$

$$= \boxed{\frac{1}{5^2} \times \frac{1}{3^2}}$$

j) $\left(\frac{3}{4}\right)^{-3}$

$$\begin{array}{l} \swarrow \quad \searrow \\ \frac{3^{-3}}{4^{-3}} \quad \left(\frac{4}{3}\right)^3 \\ \frac{\frac{1}{3^3}}{\frac{1}{4^3}} \quad \boxed{\frac{4^3}{3^3}} \\ \frac{1}{3^3} \times \frac{4^3}{1} \\ = \boxed{\frac{4^3}{3^3}} \end{array}$$