Key Ideas

- A linear inequality compares linear expressions that may not be equal.
  $x \geq -3$ means that x is greater than or equal to -3.
- Situations involving inequalities can be represented verbally, graphically, and algebraically.
  - Verbally: Use words.
  - Graphically: Use visuals, such as diagrams and graphs.
  - Algebraically: Use mathematical symbols, such as numbers, variables, operation signs, and the symbols $<$, $>$, $\leq$, and $\geq$.

A person must be under twelve years of age to qualify for a child's ticket at the movies. Let a represent the age of the person.

The values of a are less than 12.

The inequality is $a < 12$.

- An inequality with the variable on the right can be interpreted two ways.
  $8 < x$ can be read "8 is less than x." This is the same as saying "x is greater than 8."

Check Your Understanding

Communicate the Ideas

1. Consider the inequalities $x > 10$ and $x \geq 10$.

a) List three possible values for x that satisfy both inequalities. Explain how you know.

b) Identify a number that is a possible value for x in one but not both inequalities.

c) How are the possible values for inequalities involving $>$ or $<$ different than for inequalities involving $\geq$ or $\leq$? Give an example to support your answer.

2. On a number line, why do you think an open circle is used for the symbols $<$ and $>$, and a closed circle for the symbols $\leq$ and $\geq$?

3. Tiffany and Charles have each written an inequality to represent numbers that are not more than 15. Their teacher says that both are correct. Explain why.

Charles: $15 \geq x$

Tiffany: $x \leq 15$

4. Consider the inequality $x \neq 5$.

a) List at least three possible values for x.

b) How many values are not possible for x? Explain.

c) Explain how you would represent the inequality on a number line.

Practise

For help with #5 to #9, refer to Example 1 on pages 342–343.

5. Write the inequality sign that best matches each term. Use an example to help explain your choice for each.

a) at least

b) fewer than

c) maximum

d) must exceed

6. For which inequalities is 4 a possible value of x? Support your answer using two different representations.

a) $x > 3$

b) $x < 4$

c) $x > -9$

d) $x \geq 4$

7. Write a word statement to express the meaning of each inequality. Give three possible values of y.

a) $y \geq 8$

b) $y < -12$

c) $y \leq 6.4$

d) $y > -12.7$

8. At the spring ice fishing derby, only fish 32 cm or longer qualify for the prize categories.

a) Draw a number line to represent the situation.

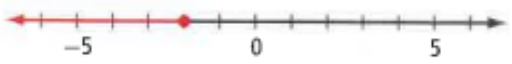
b) Write a statement to represent the sizes of fish that qualify for prizes.

For help with #9 to #12, refer to Example 2 on pages 343–344.

9. Write a word statement to express each inequality.

a)

b)



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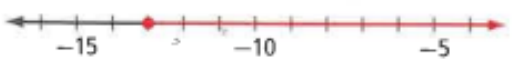


10. Express each inequality algebraically in two different ways.

a)

b)

c)

11. Sketch a number line to show each inequality.

a) $x > 3$ **b)** $x < 12$

c) $x \geq -19$ **d)** $-3 \geq x$

12. Represent each inequality graphically.

a) $y \leq 10.7$ **b)** $y \geq -5.3$

c) $y < -\frac{4}{5}$ **d)** $4.8 > x$

For help with #13 to #15, refer to Example 3 on page 345.

13. For each combination of inequalities, show the possible values for x on a number line.

a) $x > 12$ and $x < 17$

b) $x \geq -5$ and $x \leq 0$

c) $x \geq 1\frac{3}{4}$ and $x \leq 4$

d) $x < -4\frac{1}{2}$ and $x > -11$

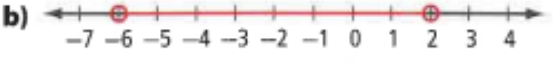
14. a) Represent the possible values for y graphically, if $y > -9.3$ and $y < -6.7$.

b) Mark any three values on the number line. For each one, explain whether it is a possible value for y.

15. Represent the values shown in red on each number line by a combination of inequalities.

a)

b)



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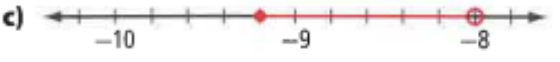


Apply

16. The manager of a clothing store has set goals for her sales staff. Express each goal algebraically.

a) The monthly total sales, m, will be a minimum of \$18 000.

b) At month end, the total time, t, spent counting store inventory will be at most 8 h.

c) The value of total daily sales, d, will be more than \$700.

17. If Emily keeps a daily balance of at least \$1500 in her bank account, she will pay no monthly fees.

a) Draw a number line to represent the situation.

b) If x represents her daily balance, write an inequality that represents the possible values for x when she will pay no fees.

18. Paul is training for a race and hopes to beat the record time. The number line represents the finishing times that will allow him to beat the record.

a) Write a statement to express the finishing times that will let Paul beat the record.

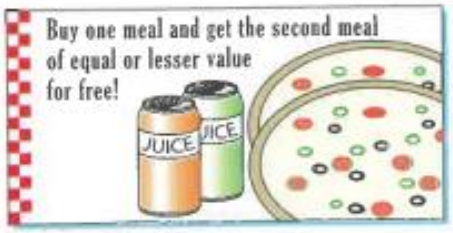
b) Express the inequality algebraically.

19. a) Develop a problem that could be represented by an inequality. Express the inequality verbally.

b) Graph the inequality.

c) Express the inequality algebraically.

20. Owen has a coupon for a restaurant.



a) Owen buys a meal for \$10.75. If m is the cost of his second meal, write an inequality to represent the possible values of m that will allow him to use the coupon.

b) Represent the inequality graphically.

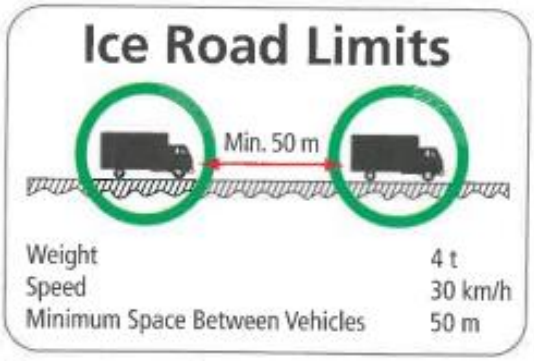
21. Shanelle is buying insurance for a car to drive to and from work. The cost of insurance will be higher if she works farther than 15 km from home.

a) Verbally express the inequality that represents the possible values for the distance for which Shanelle will have to pay more insurance.

b) Sketch a number line to represent the inequality.

22. During winter, ice roads allow access to remote places in northern communities. The ice road to Aklavik, NWT is made through the Mackenzie River Delta. The ice road to Tuktoyuktuk travels up the Mackenzie River and out onto the sea ice. Ice roads are made by flooding the existing ice on a river or lake until it reaches the required thickness.

For safety reasons, there are restrictions such as the ones shown.



Represent each restriction

a) graphically

b) algebraically

Literacy Link

A metric tonne (t) is a measurement of mass that equals 1000 kg.

Extend

23. **a)** If the inequalities $x \geq 6$ and $x \leq 6$ are both true, describe the possible values for x.

b) What would a number line showing possible values of x look like for this situation? Justify your answer.

24. Bluesky is building a wooden puzzle triangle. She has cut two sides that measure 30 cm and 80 cm, respectively. The longest side of the triangle is 80 cm. Write inequalities to represent the possible lengths for the third side of the triangle.

25. What values of x would each of the following combinations of inequalities represent? Explain verbally and show graphically.

a) $x > 4$ and $x < 7$ **b)** $x < 4$ and $x < 7$

c) $x > 4$ and $x > 7$ **d)** $x < 4$ and $x > 7$

Math Link

For safety reasons, some amusement park rides have age and height restrictions for riders.

a) Choose an amusement park ride that you have seen or design one of your own. Describe your ride.

b) For your ride, consider the safety restrictions or conditions that you might impose on riders. List at least three restrictions. Use terms of your choice.

c) Represent each restriction algebraically using a different variable for each.

d) Sketch a sign. Use words and graphics that clearly inform riders about each of your restrictions.

10. a) $x = 3.4$ **b)** $k = -63.6$ **c)** $q = -2.27$
d) $a = -5.9$
11. a) $q = 1.1$ **b)** $y = 0.071$ **c)** $n = -2.18$ **d)** $p = 0$
12. $x = -1.7$
13. a) $3(x + 1.05) = 9.83$, $x = 2.2$
14. $x = 6.76$
15. 3.3 °C
16. $d = -0.45$
17. \$2.99
18. \$37.50
19. \$6.40/skin
20. a) $h = 7.8$ cm **b)** $a = 1.3$ m
21. 15.6 cm
22. 15 years old
23. a) $x = -2.3$ **b)** $y = 4.9$ **c)** $f = 2.8$ **d)** $t = -8.98$
24. a) $d = -16.8$ **b)** $r = 3.5$ **c)** $g = 1.6$ **d)** $h = -18$
25. The longer side is 0.5 units and the shorter side is 0.25 units.
26. −15
27. 1 h 27.6 min
28. a) $n = \frac{4}{x} + 3$ **b)** $n = \frac{4}{x} + 3$
c) Divide first, because it involves fewer steps.

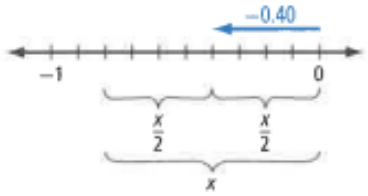
8.4 Solving Equations: $ax = b + cx$, $ax + b = cx + d$, $a(bx + c) = d(ex + f)$, pages 326–329

4. $3x + 0.15 = 2x + 0.30$, $x = 0.15$
5. $x = \$0.55$

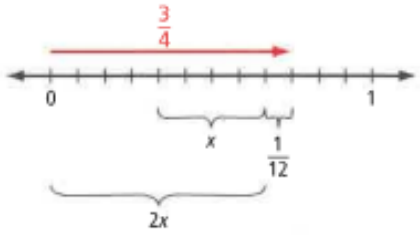
6. a) $x = 6.4$ **b)** $y = 3$ **c)** $a = -0.8$ **d)** $g = \frac{6}{7}$
7. a) $n = \frac{4}{3}$ **b)** $w = -2.2$ **c)** $p = 4.25$ **d)** $e = \frac{3}{10}$
e) $d = -18$
8. a) $k = 0.5$ **b)** $p = -21$ **c)** $u = 3$ **d)** $h = -\frac{5}{2}$
9. a) $r = -2.55$ **b)** $c = 0.4$ **c)** $k = 8$ **d)** $p = \frac{11}{3}$
10. a) $q = 0.22$ **b)** $x = -5$ **c)** $y = 0.75$ **d)** $x = \frac{5}{2}$
11. a) $s = -0.4$ **b)** $g = 7.8$ **c)** $x = 6$ **d)** $m = -2$
12. a) $c = -3.36$ **b)** $n = 3.38$ **c)** $x = 1.39$ **d)** $a = -0.17$
13. a) 19 nickels **b)** \$1.90
14. 5 weeks
15. Rectangle A: 2.5 units by 3.1 units; Rectangle B: 0.1 units by 5.5 units
16. a) $x = 1.4$ **b)** The perimeter of each triangle is 23.3 units.
17. 33 min
18. Each rectangle has an area of 13.68 square units.
19. a) 17.5 min **b)** 1.3125 km
20. $f = 180$ cm
21. 12 movies/year
22. The speed of the current is 7.07 km/h.
23. a) Yes. When the distributive property is applied to both sides of the expression, the result is two identical expressions. **b)** Yes. When the distributive property is applied to both sides of the expression, the result is two identical expressions.
24. Example: My dad always tells the same story about how hard he worked when he first came to Canada. He said he worked after school as a server in a restaurant earning \$$x$/h. One day, he worked 3.5 h and had \$1.2 in tips. Another day, he worked 4 h and had \$0.90 in tips. Both days, he earned the same amount of money. How much was my dad getting per hour?
25. Example: One video store charges a \$5 annual membership fee and \$4 per movie. Another store charges no membership fee, but \$5 per movie. How many movies per year would you have to rent for the cost to be the same at both stores? Let m represent the number of movies. $4m + 5 = 5m$, $m = 5$ movies.
26. a) $x = 17.5$ **b)** $y = -30$ **c)** $d = 2.2$ **d)** $j = 1.5$
27. a) $a = 1.125$ **b)** $s = 9$ **c)** $q = -3$ **d)** $z = -1.25$
28. $k = \frac{10 - x}{3}$
29. $n = -1.2$
30. a) $x = \frac{1}{14}$ **b)** $y = -\frac{1}{3}$

Chapter 8 Review, pages 330–331

1. D, B, A, C, C
2. opposite operation
3. distributive property
4. $x = -0.8$



5. a) $d = -\frac{1}{10}$ **b)** $y = 8.04$ **c)** $h = -17.5$ **d)** $u = 1.9$
6. a) $w = -4.7$ **b)** $k = 9.0$
7. a) $m = 43.3$ g **b)** $v = 12.5$ cm^3
8. 60 goals
9. $x = \frac{1}{3}$



10. a) She should have divided by −3. **b)** $g = 3$

11. a) $t = -14.56$ **b)** $x = 9.5$ **c)** $r = \frac{4}{3}$ **d)** $v = -\frac{20}{7}$
12. \$34.95
13. 58.6 Earth days
14. a) $e = -6.7$ **b)** $r = 1.5$ **c)** $h = -21.1$ **d)** $q = \frac{7}{8}$
15. $m = -1.97$
16. $k = 3.225$ m
17. \$21.75
18. a) $n = 15$ **b)** $f = 1.75$ **c)** $g = -1.6$ **d)** $h = -60$
e) $v = -\frac{11}{6}$
19. a) $a = 1.9$ **b)** $P = 25.2$ units
20. $w = 7.5$ cm

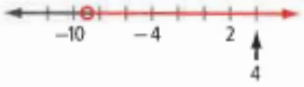
Chapter 9

9.1 Representing Inequalities, pages 347–349

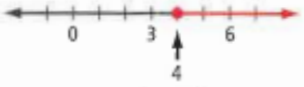
5. a) Example: $x \geq 3$ **b)** Example: $x < 7$
c) Example: $x \leq -13$ **d)** Example: $x > -1.5$
6. a) Yes; 4 is greater than 3.

b) No; 4 is not less than 4.

c) Yes; 4 is greater than −9.



d) Yes; 4 is greater than or equal to 4.



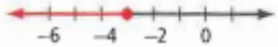
7. a) Example: All values greater than or equal to 8. Three possible values are 11, 15, and 22.
b) Example: All values less than −12. Three possible values are −14, −21.5, and −100.
c) Example: All values less than or equal to 6.4. Three possible values are 1, 3, and 6.4.
d) Example: All values that exceed −12.7. Three possible values are −11, 0, and 33.
8. a)

b) $p \geq 32$
9. a) All values greater than 4. **b)** All values less than or equal to −2. **c)** All values greater than or equal to −13.
10. a) Example: $x < 12.7$ or $12.7 > x$
b) Example: $y > 4.65$ or $4.65 < y$
c) Example: $y \leq -24.3$ or $-24.3 \geq y$
11. a)

b)

c)

d)



12. a)

b)

c)

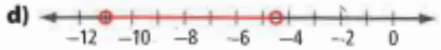
d)

13. a)

b)

c)

d)



14. a)

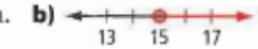
b) Example: The values of −10.0 and −9.8 are both less than −9.3, so they are not possible values. Conversely, −9.0 is larger than −9.3 so it is a possible value.

15. a) The value is greater than or equal to 20, and less than or equal to 27; $20 \leq x$ and $x \leq 27$ **b)** The value is less than 2, and greater than −6; $-6 < x$ and $x < 2$ **c)** The value is less than −8, and greater than or equal to −9.2; $-9.2 \leq x$ and $x < -8$
16. a) $m \geq 18\,000$ **b)** $t \leq 8$ **c)** $d > 700$
17. a)

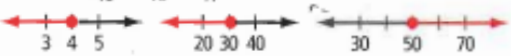
b) $x \geq 1500$
18. a) Paul will beat the record if he finishes the race in less than 41.5 s. **b)** $t < 41.5$
19. a) Example: A school environmental awareness club hopes to recycle at least 650 cans each month.
b)

c) $c \geq 650$
20. a) $m \leq 10.75$ **b)**

21. a) Shanelle will have to pay more insurance if the distance between her home and workplace is farther than 15 km. **b)**



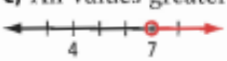
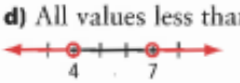
22. a)



b) $w \leq 4$; $s \leq 30$; $m \geq 50$
23. a) $x = 6$ **b)** Since the only possible value for x that satisfies both inequalities is 6, there will be a single solid dot on the number line at 6.
24. $50 < s \leq 80$
25. a) All values greater than 4 and less than 7

b) All values less than 4

c) All values greater than 7



d) All values less than 4 and greater than 7