

2.3 Integral Exponents**Exponent Law Summary**

Exponent Law	
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers	$(a^m)(a^n) = a^{m+n}$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = (a^m)(b^m)$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
Zero Exponent	$a^0 = 1, a \neq 0$

Exploring Negative Exponents:

Continue the pattern:

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{8} = \frac{1}{2^3}$

$2^{-4} = \frac{1}{16} = \frac{1}{2^4}$

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

$3^4 = 81$

$3^3 = 27$

$3^2 = 9$

$3^1 = 3$

$3^0 = 1$

$3^{-1} = \frac{1}{3}$

$3^{-2} = \frac{1}{9}$

$3^{-3} = \frac{1}{27}$

$3^{-4} = \frac{1}{81}$

$10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$

Simplify using expansion and cancellation, then simplify by exponent laws:

$\frac{4^2}{4^5} = \frac{4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{1}{4^3} = \frac{1}{64}$

$4^{2-5} = 4^{-3}$

$$\begin{aligned} \frac{(-2)^3}{(-2)^7} &= (-2)^{3-7} \\ &= (-2)^{-4} \\ &= \frac{1}{(-2)^4} = \frac{1}{16} \end{aligned}$$

Summary:

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Ex. Write each expression with positive exponents.

$$5^{-2} = \frac{1}{5^2}$$

$$\frac{1}{z^{-5}} = z^5$$

$$(xy)^{-2} = \frac{1}{(xy)^2} = \frac{1}{x^2 y^2}$$

$$x^{-5} = \frac{1}{x^5}$$

$$(4y)^{-3} = \frac{1}{(4y)^3} = \frac{1}{64y^3}$$

$$xy^{-2} = x \cdot \frac{1}{y^2} = \frac{x}{y^2}$$

$$\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^3$$

$$4y^{-3} = \frac{4}{y^3}$$

$$x^3 y^{-1} z^{-7} = \frac{x^3}{y z^7}$$

$$\frac{1}{6^{-3}} = 6^3$$

Ex. Write each product or quotient as a power with a single exponent.

$$\begin{aligned} (5^8)(5^{-3}) &= 5^{8+(-3)} \\ &= 5^5 \end{aligned}$$

$$\begin{aligned} (0.8^{-2})(0.8^{-4}) &= (0.8)^{-2+(-4)} \\ &= 0.8^{-6} = \left(\frac{8}{10}\right)^{-6} = \left(\frac{4}{5}\right)^{-6} \\ &= \frac{1}{0.8^6} = \left(\frac{5}{4}\right)^6 \end{aligned}$$

$$\begin{aligned} \frac{(2x)^3}{(2x)^{-2}} &= (2x)^{3-(-2)} \\ &= (2x)^5 \end{aligned}$$

$$\begin{aligned} (-4)^3(-4)^{-8} &= (-4)^{3+(-8)} \\ &= (-4)^{-5} \\ &= \frac{1}{(-4)^5} \end{aligned}$$

$$\begin{aligned} \frac{x^5}{x^{-3}} &= x^{5-(-3)} \\ &= x^8 \end{aligned}$$

$$\begin{aligned} \frac{q^{-1}}{q^7} &= q^{-1-7} \\ &= q^{-8} \\ &= \frac{1}{q^8} \end{aligned}$$

Ex. Write each expression as a power with a single, positive exponent. Then, evaluate where possible.

$$\begin{aligned}(4^3)^{-2} &= 4^{-6} \\ &= \frac{1}{4^6} \\ &= \boxed{\frac{1}{4096}}\end{aligned}$$

$$\begin{aligned}\left(\frac{2^4}{2^6}\right)^{-3} &= \frac{2^{-12}}{2^{-18}} \\ &= (2^{-2})^{-3} = 2^{-12 - (-18)} \\ &= 2^6 = \boxed{64}\end{aligned}$$

$$\begin{aligned}\left[\frac{(y^2)^0}{(y^{-4})^2}\right]^{-3} &= \left[\frac{y^0}{y^{-8}}\right]^{-3} \\ &= \left[\frac{1}{y^{-8}}\right]^{-3} = [y^8]^{-3} \\ &= \boxed{\frac{1}{y^{24}}} = y^{-24} = \boxed{\frac{1}{y^{24}}}\end{aligned}$$

$$\begin{aligned}[(a^{-2})(a^3)]^{-1} &= [a]^{-1} \\ &= a^{-1} \\ &= \boxed{\frac{1}{a}}\end{aligned}$$

$$\begin{aligned}\left[\left(\frac{3}{4}\right)^{-2} \left(\frac{3}{4}\right)^4\right]^{-2} &= \left[\left(\frac{3}{4}\right)^2\right]^{-2} \\ &= \left(\frac{3}{4}\right)^{-4} \\ &= \left(\frac{4}{3}\right)^4 = \boxed{\frac{256}{81}}\end{aligned}$$

Ex. It is estimated that there are 117 billion grasshoppers in an area of 39 000 km² of Saskatchewan. Approximately how many grasshoppers are there per square kilometre? Solve using exponents without a calculator.

$$\begin{aligned}\frac{117\,000\,000\,000}{39\,000} &= \frac{(117)(10^9)}{(39)(10^3)} \\ &= (3)(10^6) \\ &= 3,000,000 \text{ grasshoppers/km}^2\end{aligned}$$

