

Chapter 3: Volume

Section 3.3 - Relationship between surface area and volume

We can compare the Surface Area and Volume by using a ratio

Ratio

- a comparison of two quantities
- Can be written in three different ways
 - Example: if a fruit bowl contains 3 oranges and 5 bananas the ratio of oranges to bananas could be shown as...

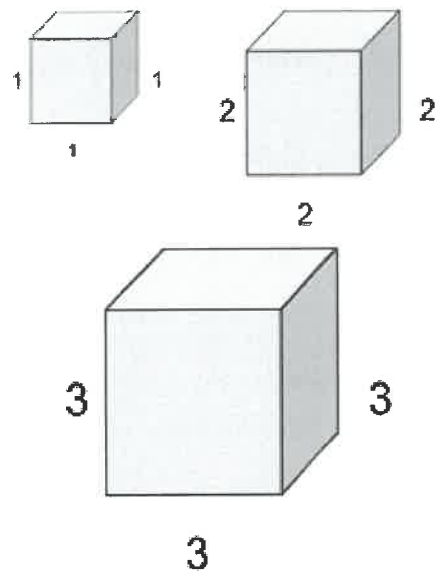
Ratio form.
oranges : bananas
3 : 5

Fraction
 $\frac{\text{oranges}}{\text{bananas}} = \frac{3}{5} = \rightarrow 0.6$

Decimal
 • a value greater than 1.
means more oranges
 • a value less than 1
means more bananas
 • a value of 1 means equal
amounts of both

Cubes

Side length (s)	Surface Area	Volume	SA:V ratio
1	6	1	$\frac{6}{1} = 6$
2	24	8	$\frac{24}{8} = 3$
3	54	27	$\frac{54}{27} = 2$
4	96	64	$\frac{96}{64} = 1.5$
5	150	125	1.2
6	216	216	1
7	294	343	0.857
8	384	512	0.75
9	486	729	0.66
10	600	1000	0.6



Calculating SA:V ratio

For side length of 1... (s=1)

$$SA:V \rightarrow \frac{SA}{V} = \frac{6}{1} = 6.$$

$$s=2 \quad SA:V \rightarrow \frac{SA}{V} = \frac{24}{8} = 3$$

$$s=3 \quad SA:V \rightarrow \frac{SA}{V} = \frac{54}{27} = 2$$

$$s=4 \quad SA:V \rightarrow \frac{SA}{V} = \frac{96}{64} = 1.5$$

$$s=5 \quad \frac{SA}{V} = \frac{150}{125} = 1.2$$

$$s=6 \quad \frac{SA}{V} = \frac{216}{216} = 1$$

$$s=7 \quad \frac{SA}{V} = \frac{294}{343} = 0.857$$

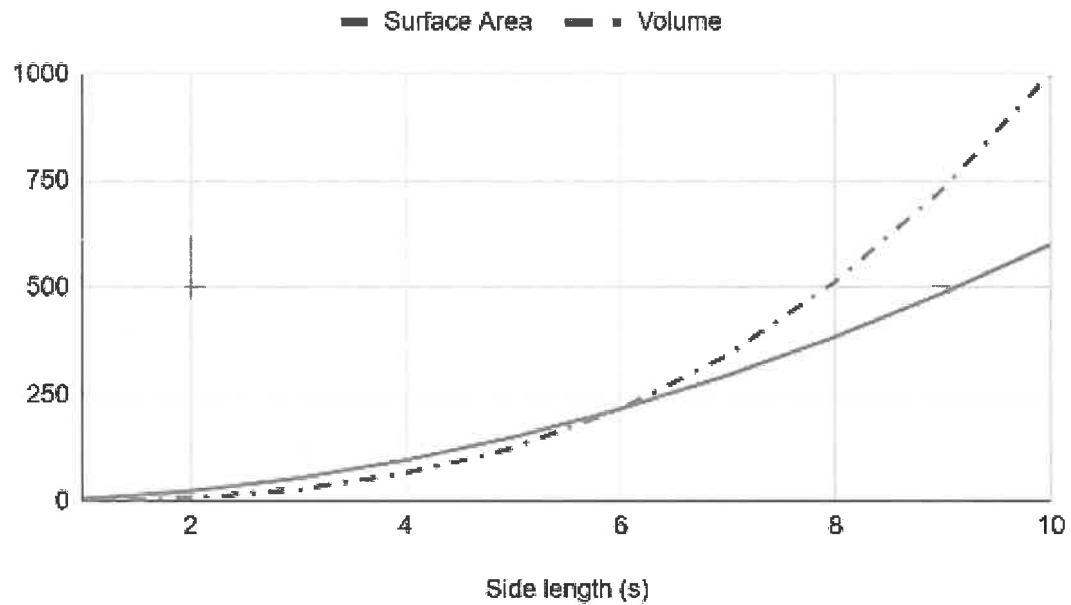
$$s=8 \quad \frac{SA}{V} = \frac{384}{512} = 0.75$$

$$s=9 \quad \frac{SA}{V} = \frac{486}{729} = 0.66$$

$$s=10 \quad \frac{SA}{V} = \frac{600}{1000} = 0.6$$

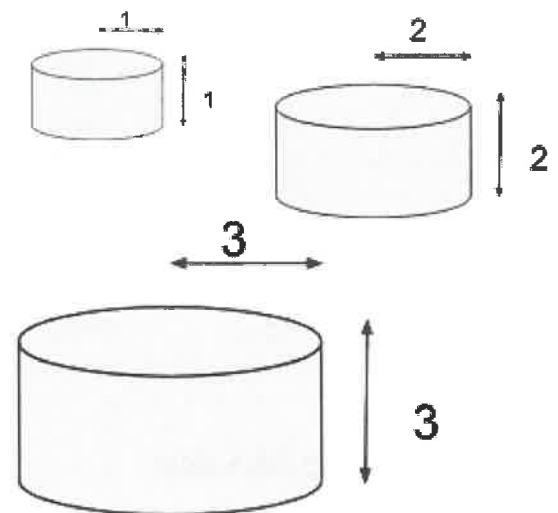
Relationship between surface area and volume of cubes

Date:



Cylinders

Radius (r)	Height (h)	Surface Area	Volume	SA:V ratio
1	1	31.42	6.28	5.00
2	2	100.53	50.27	2.00
3	3	207.35	169.65	1.22
4	4	351.86	402.12	0.88
5	5	534.07	785.40	0.68
6	6	753.98	1357.17	0.56
7	7	1011.59	2155.13	0.47
8	8	1306.90	3216.99	0.41
9	9	1639.91	4580.44	0.36
10	10	2010.62	6283.19	0.32



Calculating SA:V ratio

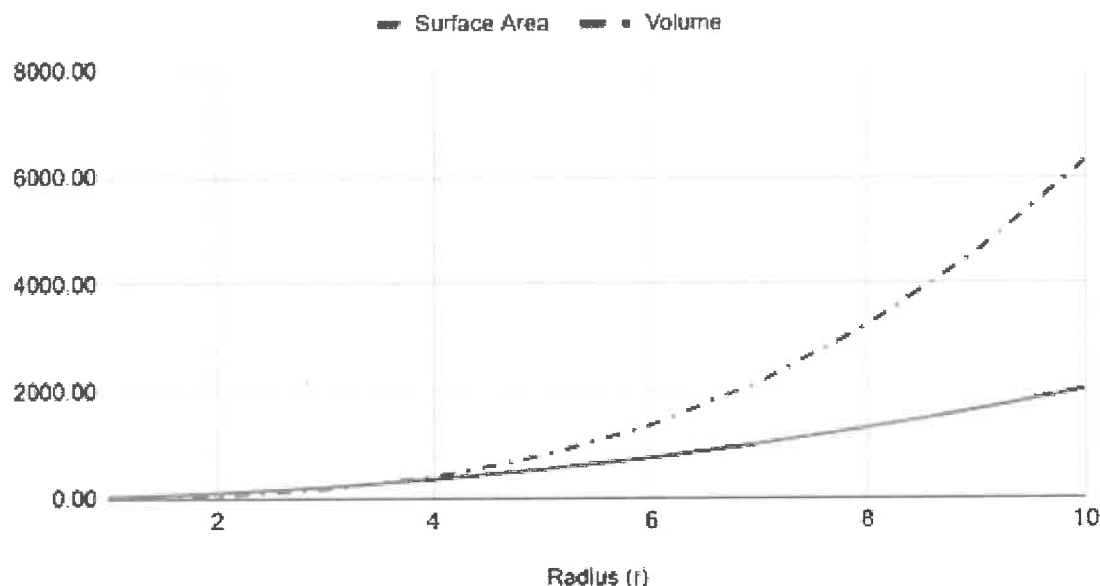
For $r=1$ $\frac{SA}{V} = \frac{31.42}{6.28} = 5.00$

Name:

Math 8

Date:

Relationship between surface area and volume of cylinders

Solving a problem using surface area to volume ratio

Suki's ice maker makes cylindrical ice cubes and crushed ice. She estimates that the ice cubes have a diameter of about 2 cm and height of 2.5 cm. The crushed ice averages about 0.5 cm in diameter and 0.5 cm in height. Use the ratio of surface area to volume to determine whether Suki should put ice cubes or crushed ice in her water bottle if she wants her water to stay cold as long as possible. (solution on page 91 of the textbook)

Cylindrical ice

$$\begin{aligned}
 d &= 2\text{ cm} \\
 r &= 1\text{ cm} \\
 h &= 2.5\text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 SA &= 2(\pi r^2) + (\pi d h) \\
 &= 2(\pi \times 1^2) + (\pi \times 2 \times 2.5) \\
 &= 6.283 + 15.708 \\
 &= 21.99\text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 V &= (\pi r^2) \times h \\
 &= \pi \times 1^2 \times 2.5 \\
 &= 7.854\text{ cm}^3
 \end{aligned}$$

$$\frac{SA}{V} = \frac{21.99}{7.854} = 2.8$$

crushed ice

$$\begin{aligned}
 d &= 0.5\text{ cm} \\
 r &= 0.25\text{ cm} \\
 h &= 0.5\text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 SA &= 2(\pi r^2) + (\pi d h) \\
 &= 2(\pi \times 0.25^2) + (\pi \times 0.5 \times 0.5) \\
 &= 0.393 + 0.785 \\
 &= 1.18\text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 V &= (\pi r^2) \times h \\
 &= \pi \times 0.25^2 \times 0.5 \\
 &= 0.098\text{ cm}^3
 \end{aligned}$$

$$\frac{SA}{V} = \frac{1.18}{0.098} = 12.04$$

The surface area to volume ratio is much greater for the crushed ice compared to the cylindrical ice. This means the crushed ice will melt faster because it has more surface area to absorb heat from the water.

Practice assignment - page 92 # 1-3, 5, choose two apply questions (challenge #10 or 11)