

Date: _____

3.5 Decomposition Factoring

Recall:

$$\begin{aligned}
 (2x-3)(x+2) &= 2x(x+2) - 3(x+2) \\
 &= 2x^2 + 4x - 3x - 6 \rightarrow \underbrace{2x^2 + 4x}_{2x(x+2)} - \underbrace{3x - 6}_{-3(x+2)} \\
 &= 2x^2 + x - 6 \rightarrow (x+2)(2x-3)
 \end{aligned}$$

$$\begin{array}{r} \otimes -12 \\ \oplus 1 \end{array}$$

$$\begin{array}{r} \oplus 1 \\ \otimes -3, 4 \end{array}$$

Decomposition FactoringFactor. $3x^2 + 8x + 4$

Step 1: Check for a GCF

$$\begin{aligned}
 &\checkmark 3x^2 + 6x + 2x + 4 \quad \begin{array}{r} \otimes 12 \\ \oplus 8 \end{array} \\
 &3x(x+2) + 2(x+2) \\
 &(x+2)(3x+2)
 \end{aligned}$$

Step 2: Find two integers with

- A product of $3 \times 4 = 12$
- A sum of 8

Step 3: Split (decompose) the middle term into two parts using the integers from step $\frac{1}{2}$

Step 4: Factor by grouping

Check?

Ex. Factor each trinomial, if possible:

a) $2x^2 + 7x - 4$ $\begin{array}{r} \otimes -8 \\ \oplus 7 \end{array}$

$$\begin{aligned}
 &\underline{2x^2 + 8x - x - 4} \\
 &2x(x+4) - 1(x+4) \\
 &(x+4)(2x-1)
 \end{aligned}$$

c) $24x^2 - 30x - 9$

$$\begin{aligned}
 &= 3(8x^2 - 10x - 3) \quad \begin{array}{r} \otimes -24 \\ \oplus -10 \end{array} \quad \begin{array}{r} -6, 4 \\ (12, +2) \end{array} \\
 &= 3(8x^2 - 12x + 2x - 3) \\
 &= 3(4x(2x-3) + 1(2x-3))
 \end{aligned}$$

b) $3x^2 + 2x + 4$ $\begin{array}{r} \otimes 12 \\ \oplus 2 \end{array}$ $\begin{array}{r} 4, 3 \\ +12 \\ 2, 6 \end{array}$

Cannot be factored

d) $6x^2 - 5xy + y^2$

$$\begin{aligned}
 &= \underline{6x^2 - 3xy - 2xy + y^2} \quad \begin{array}{r} \otimes 6 \\ \oplus -5 \end{array} \\
 &= 3x(2x-y) - y(2x-y) \quad \begin{array}{r} -3, -2 \end{array} \\
 &= (2x-y)(3x-y)
 \end{aligned}$$

e) $-3a^3b - 51a^2b - 30ab$
 $= -3ab(a^2 + 17a + 10)$

$\begin{array}{c} \textcircled{\times} 10 \\ \textcircled{+} 17 \end{array}$

cannot factor

f) $2y^2 + 7xy + 3x^2$

$= 2y^2 + 6xy + xy + 3x^2$
 $= 2y(y + 3x) + x(y + 3x)$
 $= (y + 3x)(2y + x)$

$\begin{array}{c} \textcircled{\times} 6 \\ \textcircled{+} 7 \\ \hline 6, 1 \end{array}$

Ex. Identify all values of q that allows each trinomial to be factored:

a) $2x^2 + qx + 5$

$\begin{array}{c} \textcircled{\times} 10 \\ \textcircled{+} 2 \end{array} \quad \begin{array}{l} 5 \cdot 2 \\ 10 \cdot 1 \\ -5 \cdot -2 \\ -10 \cdot -1 \end{array}$

$q = 7, 11, -7, -11$

b) $3x^2 + qxy - 4y^2$

$\begin{array}{c} \textcircled{\times} -12 \\ \textcircled{+} 2 \end{array} \quad \begin{array}{l} -6 \cdot 2 \\ -2 \cdot 6 \\ -3 \cdot 4 \\ -4 \cdot 3 \\ -1 \cdot 12 \\ -12 \cdot 1 \end{array}$

$q = \pm 4, \pm 1, \pm 11$